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Solutions of Time Fractional Mathematical Model with Effective Techniques

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Abstract

In this article, the time-fractional Clannish Random Walker's parabolic equation solutions of the traveling wave, a nonlinear partial differential equation, are analyzed using the Modified Exponential Function Model (MEFM) and the General Kudryashov Model (GKM). The solution functions of the mathematical model were obtained by a mathematical programme using two effective methods. The program was used to draw 2D and 3D contours that simulate the behaviour of this nonlinear mathematical model under appropriate parameters.

Keywords: the time fractional Clannish Random Walker's parabolic equation, modified exponential function method, generalized Kudryashov method

Zaman Kesirli Matematiksel Modelin Etkili Tekniklerle Çözümü

Öz

Bu makalede, doğrusal olmayan bir kısmi diferansiyel denklem olan Zaman Kesirli Clannish Random Walker'ın Parabolik Denklemi hareketli dalga çözümleri, Geliştirilmiş Üstel Fonksiyon Metodu (GÜFM) ve Genelleştirilmiş Kudryashov Metodu (GKM) kullanılarak analiz edilmektedir. Bu şekilde, matematiksel modelin çözüm fonksiyonları, iki etkili yöntem yardımıyla matematiksel bir program aracılığıyla elde edilmiştir. Doğrusal olmayan bu matematiksel modelin davranışını simüle eden iki boyutlu, üç boyutlu kontur grafikleri program yardımıyla uygun parametreler altında çizilmiştir.

Anahtar Kelimeler: zaman kesirli Clannish Random Walker'ın parabolik denklemi, geliştirilmiş üstel fonksiyon metodu, genelleştirilmiş Kudryashov metodu

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Introduction

Nonlinear partial differential equations (NPDEs) serve as mathematical models that find applications in various fields of engineering and healthcare. For instance, they have been employed in electronic engineering to describe phenomena such as the tension in water pipes (Ergün & Amirov, 2022; Ergun 2020a; Ergun, 2020b) and in healthcare to model the progression of events like the Covid-19 pandemic. These examples demonstrate the wide-ranging utility of these nonlinear mathematical models in explaining phenomena across different aspects of life. As a result, researchers have developed various methods and techniques to obtain solutions for these critical equation models. Some of these methods include the Bernoulli sub-equation method (Baskonus et al., 2015; Zheng, 2014; Zhou, 2014), the G'/G method (Ebadi & Biswas, 2011; Kudryashov, 2010; Zayed & Gepreel, 2009), the (m+1/G')-Expansion method (Bulut et al., 2021), the sine-Gordon method (Akbar et al., 2021; Yel et al., 2017), and the extended trial equation method (Ekici et al., 2017a; Ekici et al., 2017b). One such mathematical model worth noting is the time-fractional Clannish Random Walker's Parabolic (CRWP) equation, which exhibits particular behaviors and characteristics (Siddique et al., 2022).

$$D_t^{\beta} u + s u_x + q u u_x + r u_{xx} = 0. \tag{1}$$

β, the beta derivative order of the above mathematical model, is in the range $\beta \in (0,1]$.

The article follows the following template from this section onwards: In the subsequent section following the introduction, we delve into the definition and properties of the fractional derivative within the mathematical model under investigation. Following this, we provide a comprehensive exposition of the Modified Exponential Function Method (MEFM) and the Generalized Kudryashov Method (GKM), both of which will be employed to derive the traveling wave solutions for the nonlinear mathematical model. The solution functions of the nonlinear fractional mathematical model are subsequently acquired by applying these methods, and the concluding section consolidates all the results and commentary presented within this article.

Analysis of Beta Derivative

Definition: The definition of the beta derivative is given below (Ghanbari, 2019),

$$D_x^{\beta}\left\{ f(x) \right\} = \lim_{\varepsilon \to 0} \frac{f\left(x + \varepsilon \left(x + \frac{1}{\Gamma(\beta)} \right)^{1-\beta} \right) - f(x)}{\varepsilon}, \ \beta \in (0,1]. \tag{2}$$

The most important factor in choosing the mathematical model that is the subject of this article is the ability to use the properties of the beta derivative rule. Accordingly, the properties of the beta derivative are as follows:

i) $$D_x^{\beta}\left\{ a\,g(x) + b\,h(x) \right\} = a D_x^{\beta}\left\{ g(x) \right\} + b D_x^{\beta}\left\{ h(x) \right\}, \forall a,b \in R. \tag{3}$$

ii) $$D_x^{\beta}\left\{ g(x) h(x) \right\} = h(x) D_x^{\beta}\left\{ g(x) \right\} + g(x) D_x^{\beta}\left\{ h(x) \right\}. \tag{4}$$

iii) $$D_x^{\beta}\left\{ \frac{g(x)}{h(x)} \right\} = \frac{h(x) D_x^{\beta}\left\{ g(x) \right\} - g(x) D_x^{\beta}\left\{ h(x) \right\}}{h^2(x)}, h(x) \neq 0. \tag{5}$$

Method

The Modified Exponential Function Method: Analyzing Its Processing Steps for Effective Application

In this section, we will provide information about the Modified Exponential Function Method (MEFM). We will discuss both the solution function of the mathematical model investigated in this study and the general form of the nonlinear partial differential equation based on the derivatives of this function. Here they are: Solution function of the mathematical model investigated in this study. General form of the nonlinear partial differential equation derived from the derivatives of this function.

$$P\left(u, D_t^{\beta}u, u_x, u_{xx}, uu_x, \cdots\right) = 0. \tag{6}$$

We introduce the following travelling transform based on the beta derivative variables in the nonlinear mathematical model to simplify the nonlinear partial differential equation shown in the figure above, which has multiple derivative variables, into a single variable.

$$U\left(x,t\right) = U\left(\xi\right), \xi = x - \frac{c}{\beta}\left(t + \frac{1}{\Gamma\left[\beta\right]}\right). \tag{7}$$

The derivative terms required in equation (6) are obtained from the traveling wave transform in equation (7) and then substituted to derive the general form of the following nonlinear ordinary differential equation,

$$N\left(U, U^2, U', U'', \cdots\right) = 0. \tag{8}$$

The assumed solution function that is expected to yield the nonlinear mathematical model discussed in the paper is as follows:

$$U(\xi) = \frac{\sum_{j=0}^{n} A_j [e^{-\vartheta(\xi)}]^j}{\sum_{i=0}^{m} B_i [e^{-\vartheta(\xi)}]^i} = \frac{A_0 + A_1 e^{-\vartheta} + \ldots + A_n e^{-n\vartheta}}{B_0 + B_1 e^{-\vartheta} + \ldots + B_m e^{-m\vartheta}}, \tag{9}$$

where $A_j, B_i, (0 \le j \le n, 0 \le i \le m)$ are any constants. The derivative terms required in equation (8) are obtained and substituted in equation (9). However, while obtaining these derivative terms, the derivatives of the ϑ function in the powers of the exponential functions in equation (8) are needed. Therefore, the following equation is used,

$$\vartheta'(\xi) = e^{-\vartheta(\xi)} + \mu e^{\vartheta(\xi)} + \lambda. \tag{10}$$

Equation (10) is integrated according to ξ. In this order, the following states are obtained according to the values of λ and μ in the equation (He, 2006):

Family 1: $\mu \neq 0$ and $\lambda^2 - 4\mu > 0$,

$$\vartheta(\xi) = ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \tag{11}$$

Family 2: $\mu \neq 0$ and $\lambda^2 - 4\mu < 0$,

$$\vartheta(\xi) = ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \tag{12}$$

Family 3: $\mu = 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu > 0$,

$$\vartheta(\xi) = -ln(\frac{\lambda}{e^{\lambda(\xi+E)} - 1}). \tag{13}$$

Family 4: $\mu \neq 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu = 0$,

$$\vartheta(\xi) = ln(-\frac{2\lambda(\xi+E)+4}{\lambda^2(\xi+E)}). \tag{14}$$

Family 5: $\mu = 0$, $\lambda = 0$ and $\lambda^2 - 4\mu = 0$,

$$\vartheta(\xi) = ln(\xi + E), \tag{15}$$

where E, λ, μ are coefficients.

According to the conditions obtained above, the ϑ in powers of exponential functions are found in the solution function considered as assumption (8). Another crucial step is to define the boundaries of the solution function. This is accomplished by establishing a relationship between the boundary terms, which involves balancing the highest order derivative term and the highest order nonlinear term within the nonlinear ordinary differential equation. Subsequently, arbitrary values are assigned to these parameters to delineate the boundaries of the solution function (Equation 8). Following this, the solution function is conjectured, and the requisite derivative terms within the nonlinear ordinary differential equation are then substituted. When this equation is arranged in terms of powers of $e^{\vartheta(\xi)}$, a system of algebraic equations is obtained. Following this procedure, we ascertain the coefficients associated with the solution function (8). Ultimately, we verify that the solution function, resulting from all these operations, satisfies the nonlinear ordinary differential equation (NODE) first, and subsequently, the nonlinear partial differential equation. We also generate graphical representations of the solution function with the assistance of specialized software.

Analyzing the Generalized Kudryashov Method for Solving Nonlinear Differential Equations

In this article, we will conduct a detailed analysis of the Generalized Kudryashov Method (GKM), which is another approach for obtaining solution functions of nonlinear mathematical models. To a certain extent, the procedures of these methods share similarities. Similar to the previously described method, the general form of the nonlinear partial differential equation under investigation in this paper is as follows:

$$P\left(u, D_t^{\beta}u, u_x, u_{xx}, uu_x, \cdots\right) = 0. \tag{16}$$

In order to simplify the solution of this nonlinear partial differential equation (NPDE), we employ the following traveling wave transform to reduce the number of derivative variables to a single variable:

$$U\left(x,t\right) = U\left(\xi\right),\ \xi = x - \frac{c}{\beta}\left(t + \frac{1}{\Gamma\left[\beta\right]}\right). \tag{17}$$

By applying the nonlinear mathematical model to this traveling wave transformation, the general form of the univariate nonlinear ordinary differential equation depending on ξ is obtained,

$$N\left(U, U^2, U', U'', \cdots\right) = 0. \tag{18}$$

In this method, similar to the previously described approach, we assume the solution function, which is expected to satisfy both the nonlinear ordinary differential equation (NODE) and the nonlinear partial differential equation (NPDE), as follows:

$$U(\xi)=\frac{\sum_{j=0}^{n}a_j[Q]^j}{\sum_{i=0}^{m}b_i[Q]^i}=\frac{a_0+a_1Q+a_2Q^2+\cdots+a_nQ^n}{b_0+b_1Q+b_2Q^2+\cdots+b_mQ^m}, \tag{19}$$

where $a_j, b_i, (0\le j\le n,\ 0\le i\le m)$ are any constants. Using the solution function (19), the derivative terms required in the nonlinear ordinary differential equation are arranged. In the meantime, the derivatives of the Q function also appear. Therefore, this term, which is one of the components of the solution function, is treated as follows (Kaplan, 2021):

$$Q'=Q^2-Q. \tag{20}$$

After determining the Q in the solution function, we start the process of bounding the solution function. It does so through the application of the balancing principle in the context of nonlinear ordinary differential equation. This principle involves balancing the term that contains the derivative of the highest order and the term of the highest order within the equation. Through the assignment of appropriate values to these parameters, we determine the bounds of the solution function. The nonlinear ordinary differential equation is then solved by substituting the solution function and its derivatives. This equation is organized according to the powers of Q, and a system of algebraic equations consisting of the coefficients in the solution function is obtained. The coefficients obtained from the solution of this system are then applied to the solution function. A subsequent process of verification is carried out to ensure that this solution function satisfies first the nonlinear ordinary differential equation (NODE) and then the nonlinear partial differential equation (NPDE). In addition, graphical representations illustrating the behaviour of the NPDE are generated using the program.

Application

Solutions of Nonlinear Mathematical Model with Modified Exponential Function Method (MEFM)

In this part of the paper, to obtain the solution functions of the time-fractional clannish random walker parabolic equation and to determine the graphs according to the appropriate parameters, the modified exponential function method introduced in detail above is used. First, the following nonlinear ordinary differential equation model of equation (1) is obtained when the derivative terms in the nonlinear mathematical model (1) are obtained from equation (7) and replaced:

$$2(s-c)U+qU^2+2rU'=0. \tag{21}$$

By balancing the highest order derivative term U^2 and the highest order nonlinear term U' in equation (21), the following relation is obtained.

$$2n-2m=n-m+1\Rightarrow n=m+1. \tag{22}$$

By means of relation (22), the boundaries of the solution function assumed to satisfy the nonlinear partial differential equation will be determined. In this process, $n=2$ is obtained for $m=1$, taking into account the technological possibilities studied. Then, the model accepted as the solution function is obtained as follows:

$$U(\xi)=\frac{A_0+A_1e^{-\vartheta}+A_2e^{-2\vartheta}}{B_0+B_1e^{-\vartheta}}. \tag{23}$$

In equation (21), all necessary function models are obtained from equation (22) and substituted. The resulting equation is then organized according to e^{ϑ} to obtain the algebraic system of equations consisting of coefficients.

Case 1.

$$A_0 = 0, A_1 = \frac{1}{2}\left(\lambda - \sqrt{\lambda^2 - 4\mu}\right)A_2, B_0 = 0, B_1 = \frac{qA_2}{2r}, c = s - r\sqrt{\lambda^2 - 4\mu}. \tag{24}$$

The preferred coefficients for this case are substituted in equation (23). Then, the selected cases of the ϑ function under the following conditions are substituted into the solution function and the mathematical model that satisfies the nonlinear partial differential equation is obtained.

Family 1: $\mu \neq 0$ and $\lambda^2 - 4\mu > 0$

$$U_{1,1}(x,t) = \frac{r\left(\lambda\left(-\lambda + \sqrt{\lambda^2 - 4\mu}\right) + 4\mu\right)\left(-1 + \tanh\left[\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\left(E + \xi\right)\right]\right)}{q\left(\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left[\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\left(E + \xi\right)\right]\right)}, \tag{25}$$

where, $\xi = x - \frac{c}{\beta}\left(t + \frac{1}{\Gamma[\beta]}\right)$.

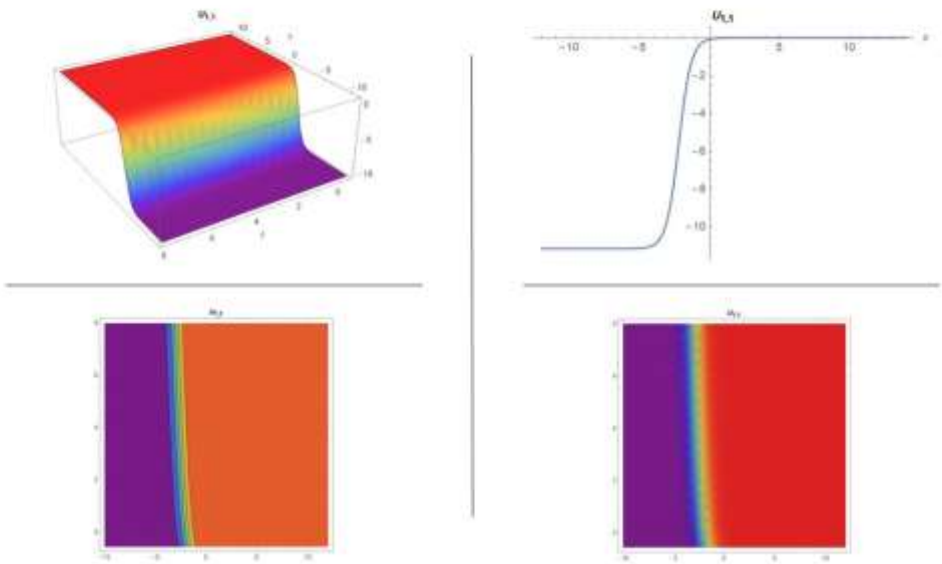

Figure 1. Display of Effective Graphs Showing the Behavior of Equation (25) as a Function of the Parameters $\lambda = 3, \mu = 1, A_2 = 0.5, q = 0.1, r = 0.25, s = 0.32, A_0 = 0, A_1 = 0.190983, B_0 = 0, B_1 = 0.1, c = -0.239017, \beta = 0.5, E = 0.75, t = 1.$

Family 2: If, $\mu \neq 0$ and $\lambda^2 - 4\mu < 0$

$$U_{1,2}(x,t) = \frac{r\left(\lambda^2 - \lambda\sqrt{\lambda^2 - 4\mu} - 4\mu + \left(\sqrt{-\left(\lambda^2 - 4\mu\right)^2} - \lambda\sqrt{-\lambda^2 + 4\mu}\right)\tan[\omega]\right)}{q\left(\lambda - \sqrt{-\lambda^2 + 4\mu}\tan[\omega]\right)}, \tag{26}$$

where, $\omega = \frac{1}{2}\sqrt{-\lambda^2 + 4\mu}\left(E + \xi\right).$

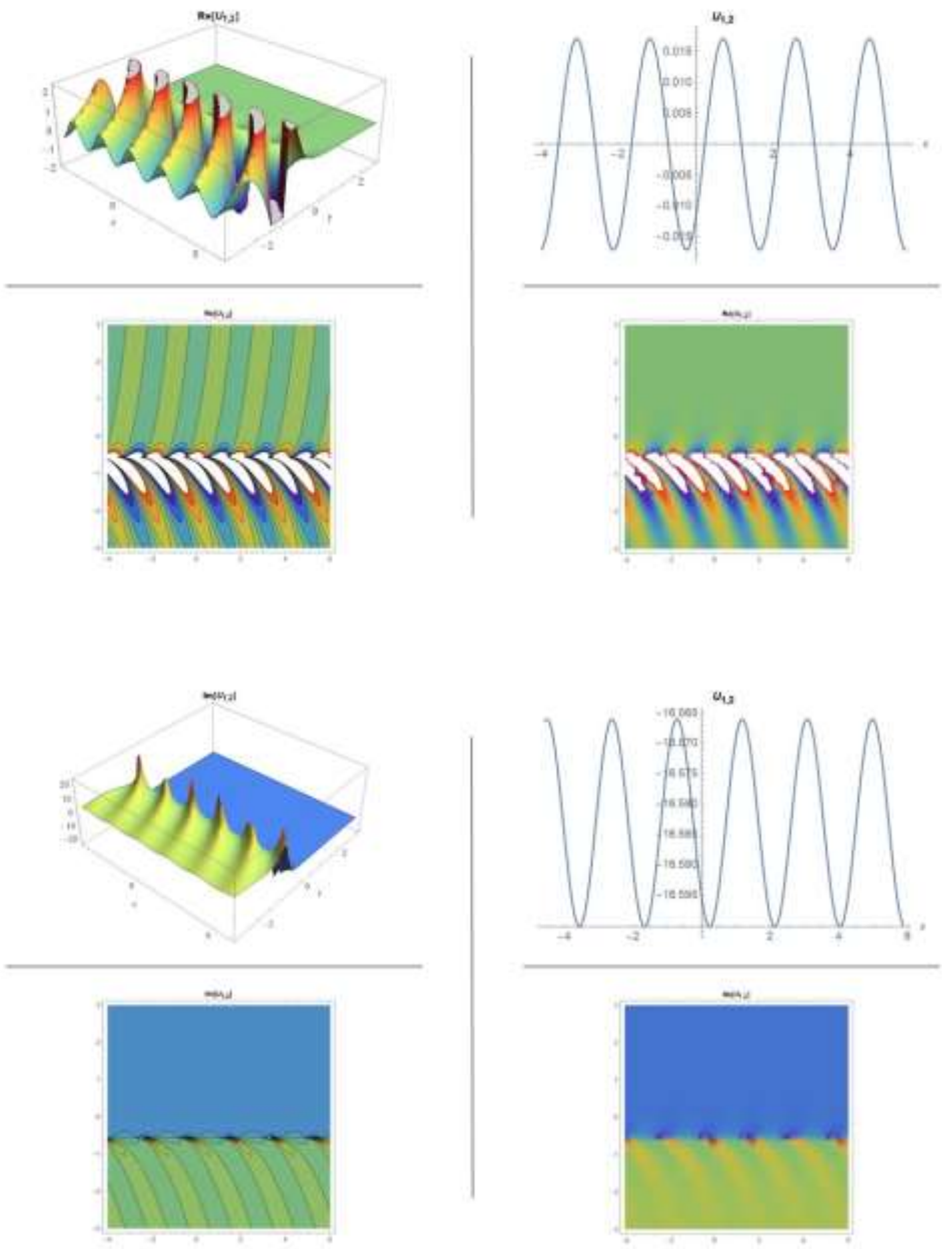

Figure 2. Display of Effective Graphs Showing the Behavior of Equation (26) as a Function of the Parameters $\lambda = 1, \mu = 3, A_2 = 0.5, q = 0.1, r = 0.25, s = 0.32, A_0 = 0, A_1 = 0.25 - 0.829156i, B_0 = 0, B_1 = 0.1, c = 0.32 - 0.829156i,$ $\beta = 0.5, E = 0.75, t = 1.$

Family 3: $\mu = 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu > 0$

$$U_{1,3}\left(x,t\right) = \frac{r\left(-\sqrt{\lambda^2} + \lambda \coth\left[\frac{1}{2}\lambda\left(E + \xi\right)\right]\right)}{q}. \tag{27}$$

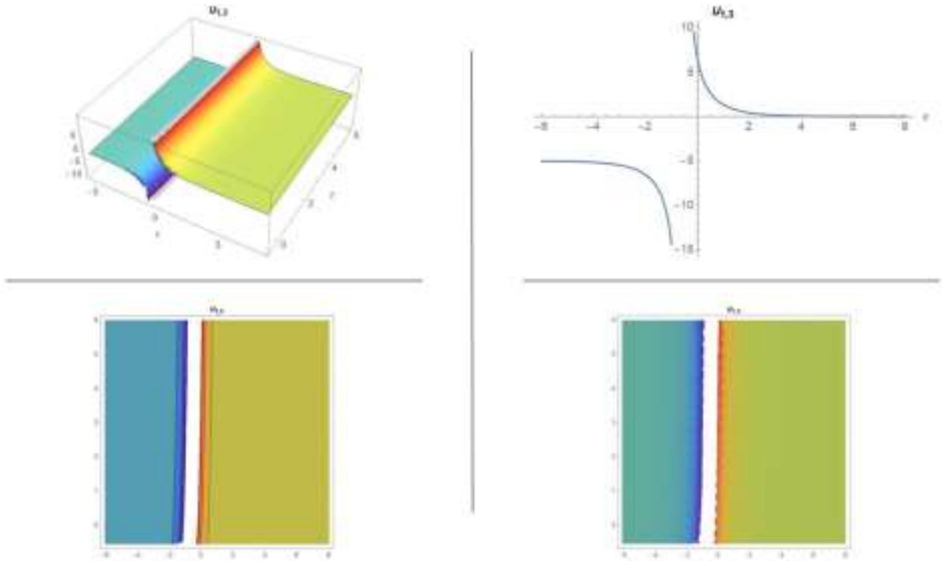

Figure 3. Display of Effective Graphs Showing the Behavior of Equation (27) as a Function of the Parameters $\lambda = 1, \mu = 0, A_2 = 0.5, q = 0.1, r = 0.25, s = 0.32, A_0 = 0, A_1 = 0, B_0 = 0, B_1 = 0.1, c = 0.07, \beta = 0.5, E = 0.75, t = 1.$

Family 4: When, $\mu \neq 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu = 0$

$$U_{1,4}(x,t) = \frac{r\left(-\sqrt{\lambda^2 - 4\mu} + \dfrac{2\lambda}{2 + \lambda(E + \xi)}\right)}{q}. \tag{28}$$

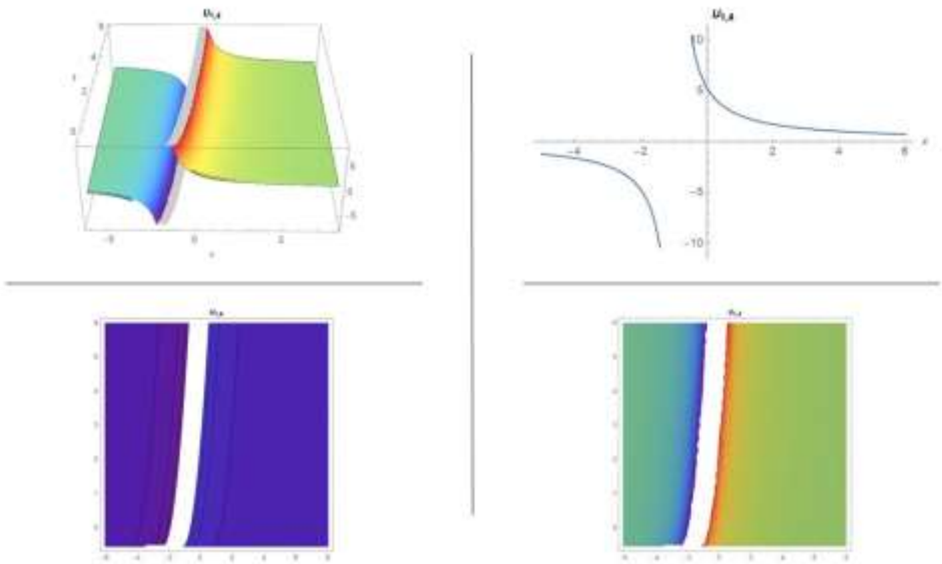

Figure 4. Display of Effective Graphs Showing the Behavior of Equation (28) as a Function of the Parameters $\lambda = 2, \mu = 1, A_2 = 0.5, q = 0.1, r = 0.25, s = 0.32, A_0 = 0, A_1 = 0.5, B_0 = 0, B_1 = 0.1, c = 0.32, \beta = 0.5, E = 0.75, t = 1.$

Family 5: $\mu = 0$, $\lambda = 0$ and $\lambda^2 - 4\mu = 0$

$$U_{1,5}(x,t) = \frac{2r}{q(E+\xi)}. \tag{29}$$

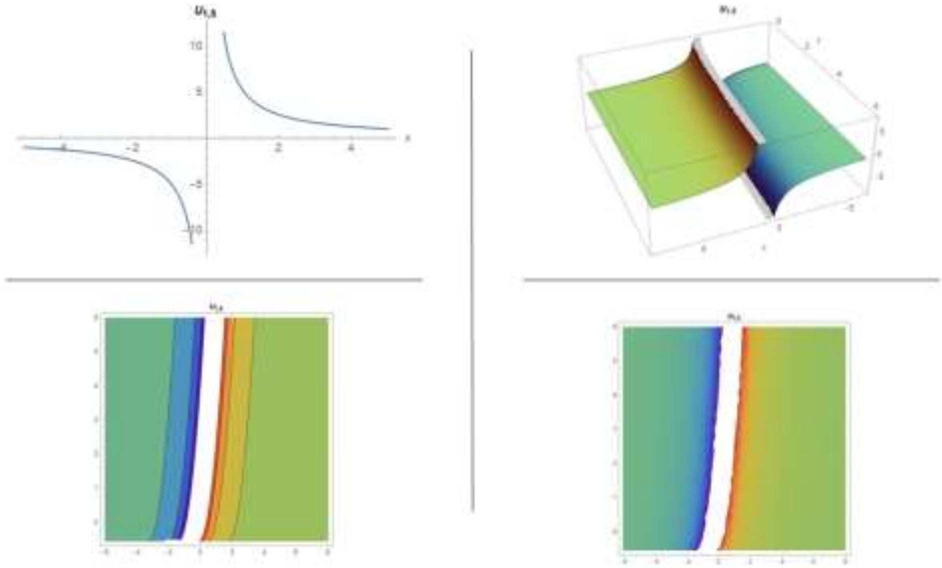

Figure 5. Display of Effective Graphs Showing the Behavior of Equation (29) as a Function of the Parameters $\lambda = 0, \mu = 0, A_2 = 0.5, q = 0.1, r = 0.25, s = 0.32, A_0 = 0, A_1 = 0, B_0 = 0, B_1 = 0.1, c = 0.32, \beta = 0.5, E = 0.75, t = 1.$

Case 2.

$$A_0 = 0, A_1 = \frac{1}{2}\left(\lambda + \sqrt{\lambda^2 - 4\mu}\right)A_2, B_0 = 0, B_1 = \frac{qA_2}{2r}, c = s + r\sqrt{\lambda^2 - 4\mu}. \tag{30}$$

The processes of obtaining the solution functions of the nonlinear mathematical model according to the second of the coefficient states get by solving the algebraic equation system are as follows.

Family 1: $\mu \neq 0$ and $\lambda^2 - 4\mu > 0$

$$U_{2,1}(x,t) = \frac{r\left(\lambda\left(\lambda + \sqrt{\lambda^2 - 4\mu}\right) - 4\mu\right)\left(1 + \tanh\left[\frac{1}{2}\sqrt{\lambda^2 - 4\mu}(E+\xi)\right]\right)}{q\left(\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left[\frac{1}{2}\sqrt{\lambda^2 - 4\mu}(E+\xi)\right]\right)}, \tag{31}$$

where, $\xi = x - \frac{c}{\beta}\left(t + \frac{1}{\Gamma[\beta]}\right).$

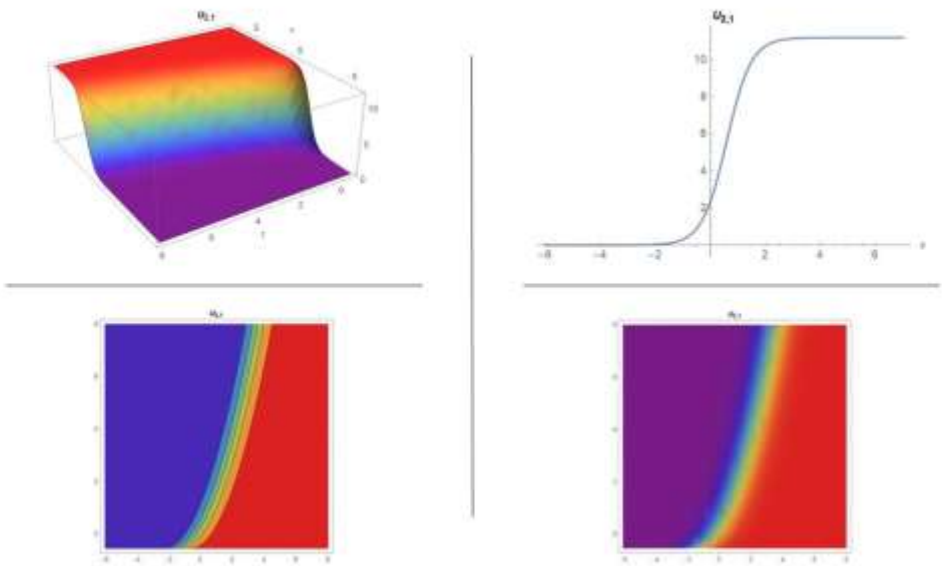

Figure 6. Display of Effective Graphs Showing the Behavior of Equation (31) as a Function of the Parameters $\lambda=3, \mu=1, A_2=0.5, q=0.1, r=0.25, s=0.32, A_0=0, A_1=1.30902, B_0=0, B_1=0.1, c=0.879017, \beta=0.5,$ $E=0.75, t=1.$

Family 2: $\mu\neq 0$ and $\lambda^2-4\mu<0$

$$U_{2,2}(x,t)=\frac{r\left(\lambda\left(\lambda+\sqrt{\lambda^2-4\mu}\right)-4\mu-\left(\sqrt{-\left(\lambda^2-4\mu\right)^2}+\lambda\sqrt{-\lambda^2+4\mu}\right)\tan[\omega]\right)}{q\left(\lambda-\sqrt{-\lambda^2+4\mu}\tan[\omega]\right)}, \quad (32)$$

where, $\omega=\frac{1}{2}\sqrt{-\lambda^2+4\mu}\left(E+\xi\right).$

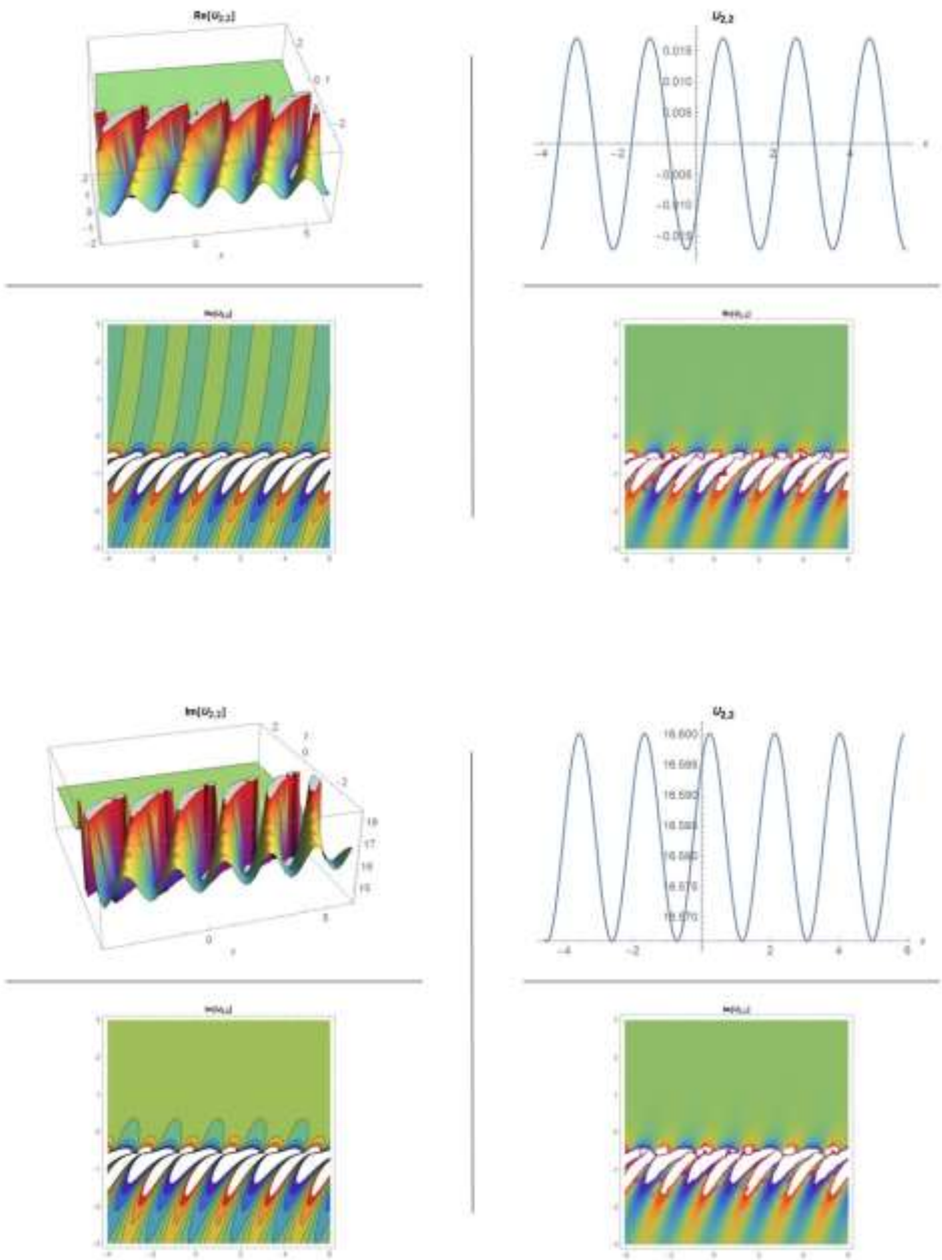

Figure 7. Display of Effective Graphs Showing the Behavior of Equation (32) as a Function of the Parameters $\lambda = 1, \mu = 3, A_2 = 0.5, q = 0.1, r = 0.25, s = 0.32, A_0 = 0, A_1 = 0.25 + 0.829156i, B_0 = 0, B_1 = 0.1, c = 0.32 + 0.829156i,$ $\beta = 0.5, E = 0.75, t = 1.$

Family 3: $\mu = 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu > 0$

$$U_{2,3}(x,t) = \frac{r\left(\sqrt{\lambda^2} + \lambda \coth\left[\frac{1}{2}\lambda(E+\xi)\right]\right)}{q}. \tag{33}$$

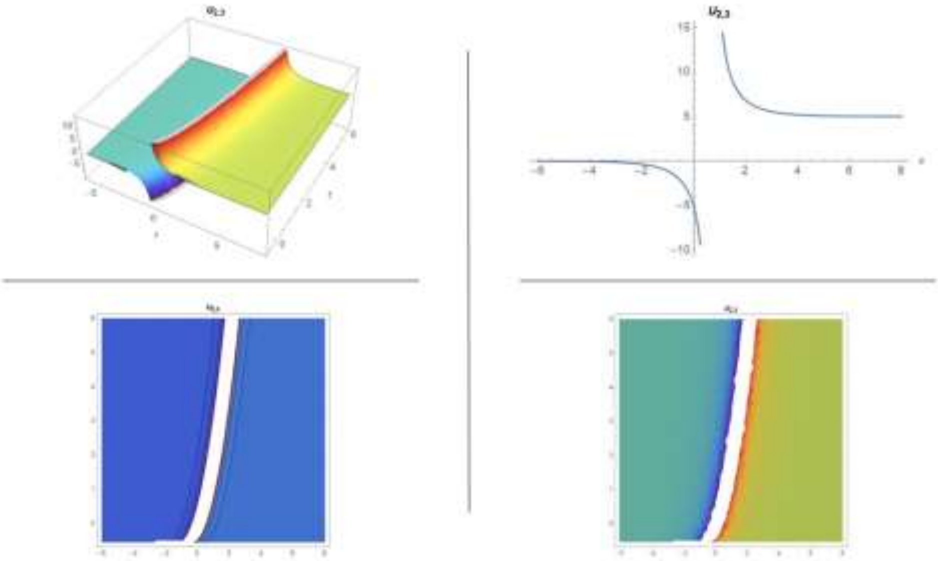

Figure 8. Display of Effective Graphs Showing the Behavior of Equation (33) as a Function of the Parameters $\lambda=1,\mu=0,A_2=0.5,q=0.1,r=0.25,s=0.32,A_0=0,A_1=0.5,B_0=0,B_1=0.1,c=0.57,\beta=0.5,E=0.75,t=1.$

Family 4: $\mu\neq 0$, $\lambda\neq 0$ and $\lambda^2-4\mu=0,$

$$U_{2,4}(x,t)=\frac{r\left(\sqrt{\lambda^2-4\mu}+\dfrac{2\lambda}{2+\lambda(E+\xi)}\right)}{q}. \tag{34}$$

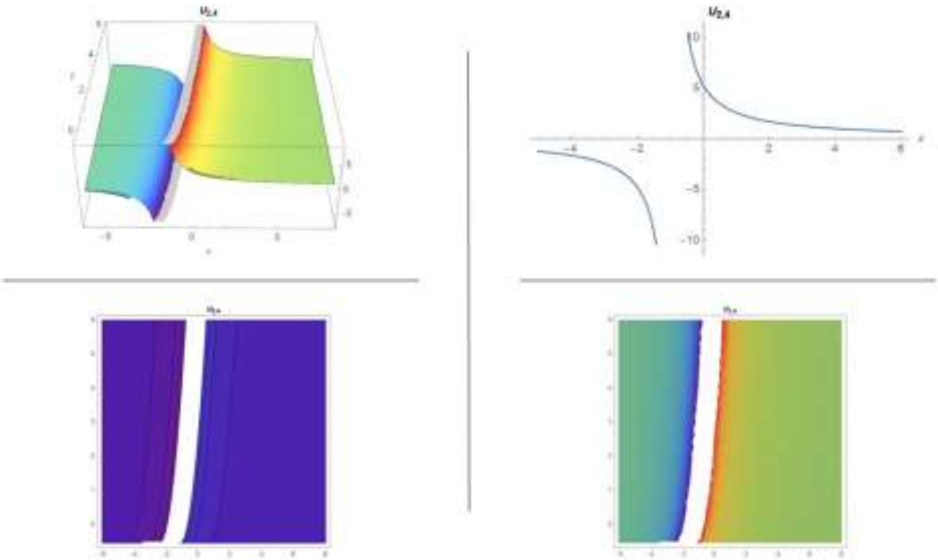

Figure 9. Display of Effective Graphs Showing the Behavior of Equation (34) as a Function of the Parameters $\lambda=2,\mu=1,A_2=0.5,q=0.1,r=0.25,s=0.32,A_0=0,A_1=0.5,B_0=0,B_1=0.1,c=0.32,\beta=0.5,E=0.75,t=1.$

Family 5: $\mu=0$, $\lambda=0$ and $\lambda^2-4\mu=0,$

$$U_{2,5}(x,t)=\frac{2r}{q(E+\xi)}. \tag{35}$$

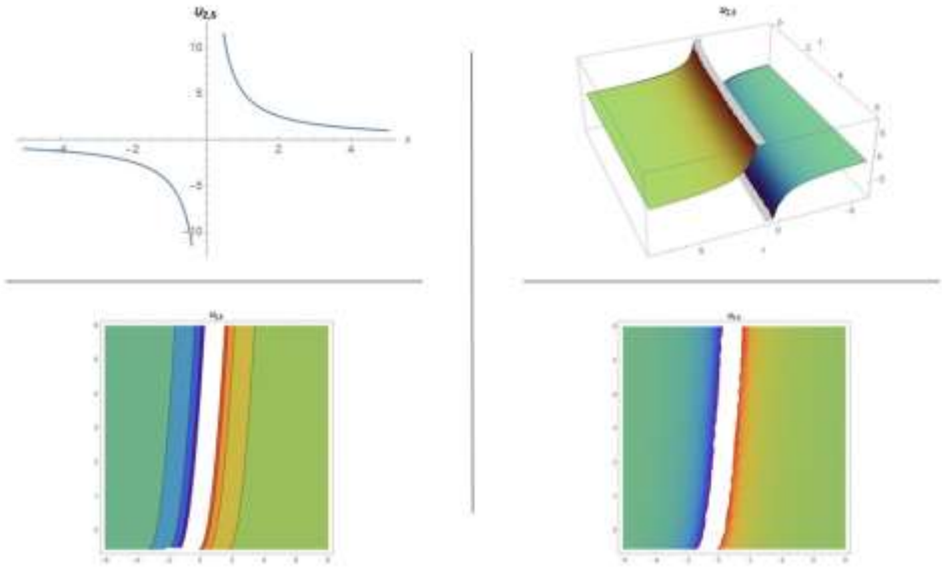

Figure 10. Display of Effective Graphs Showing the Behavior of Equation (35) as a Function of the Parameters $\lambda=0,\mu=0,A_2=0.5,q=0.1,r=0.25,s=0.32,A_0=0,A_1=0,B_0=0,B_1=0.1,c=0.32,\beta=0.5,E=0.75,t=1.$

Analysis of Wave Solutions of Mathematical Model with the Generalized Kudryashov Method (GKM)

In this part of the article, the steps of the GKM, which is the second method in the article for obtaining the traveling wave solutions of the the Time Fractional Clannish Random Walker's Parabolic Equation will be given. As mentioned earlier, where the two methods used in the paper diverge is in the difference of the solution functions taken as assumptions. According to this method, the solution function is as follows.

$$U(\xi)=\frac{\sum_{j=0}^{2}a_j[Q]^j}{\sum_{i=0}^{1}b_i[Q]^i}=\frac{a_0+a_1Q+a_2Q^2}{b_0+b_1Q}, \tag{36}$$

where $a_j, b_i, (0\le j\le n,\ 0\le i\le m)$ are any constants. Then, the derivative terms required in the NODE are get from the solution function accepted as this assumption and written in their place. The equation found is arranged according to the powers of Q.

Case 1.

$$a_0=0,\ a_2=-a_1,\ b_0=-\frac{q\,a_1}{2r}b_1,\ c=-r+s. \tag{37}$$

The equations to be written in place of Q with the solution of equation (20) are as follows.

$$Q=\left(\frac{1}{2}-\frac{1}{2}\tanh\left[\frac{\xi}{2}\right]\right),Q=\left(\frac{1}{2}-\frac{1}{2}\coth\left[\frac{\xi}{2}\right]\right). \tag{38}$$

By using the coefficients obtained above and these equations where Q is equal, the solution functions of equation (1) are obtained as follows.

$$U_{3,1}(x,t)=\left(\frac{r\left(-1+\tanh\left[\frac{1}{2}\xi\right]\right)}{q}\right), \tag{39}$$

where, $\xi = x-\frac{c}{\beta}\left(t+\frac{1}{\Gamma[\beta]}\right)$.

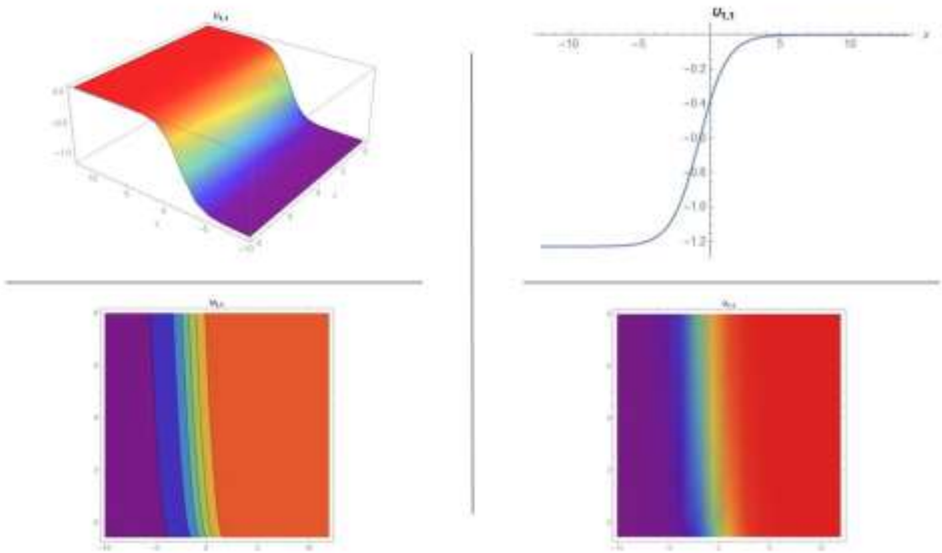

Figure 11. Display of Effective Graphs Showing the Behavior of Equation (39) as a Function of the Parameters $a_1 = 0.24, q = 1.22, r = 0.75, s = 0.44, a_0 = 0, a_2 = -0.24, b_0 = -0.1952, b_1 = 0.1952, c = -0.31, \beta = 0.5, E = 0.75, t = 1.$

$$U_{3,2}(x,t)=\left(\frac{r\left(-1+\coth\left[\frac{1}{2}\xi\right]\right)}{q}\right). \tag{40}$$

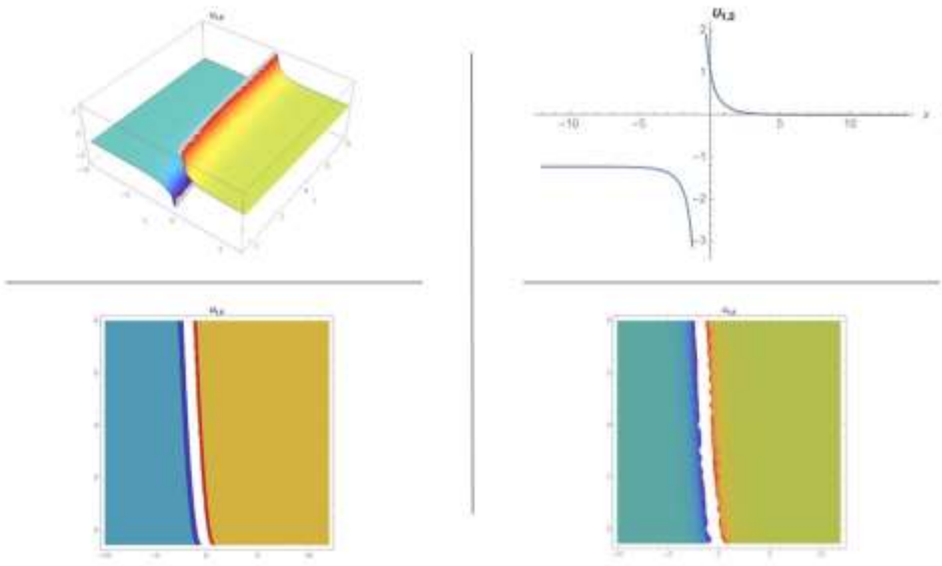

Figure 12. Display of Effective Graphs Showing the Behavior of Equation (40) as a Function of the Parameters $a_1 = 0.24, q = 1.22, r = 0.75, s = 0.44, a_0 = 0, a_2 = -0.24, b_0 = -0.1952, b_1 = 0.1952, c = -0.31, \beta = 0.5, E = 0.75, t = 1.$

Case 2.

$$a_0 = \frac{4rb_0}{q},\ a_1 = -\frac{8rb_0}{q},\ a_2 = \frac{4rb_0}{q},\ b_1 = -2b_0,\ c = 2r + s. \tag{41}$$

$$U_{4,1}(x,t) = \left(\frac{r\left(2 + \coth\left[\frac{1}{2}\xi\right] + \tanh\left[\frac{1}{2}\xi\right] \right)}{q} \right). \tag{42}$$

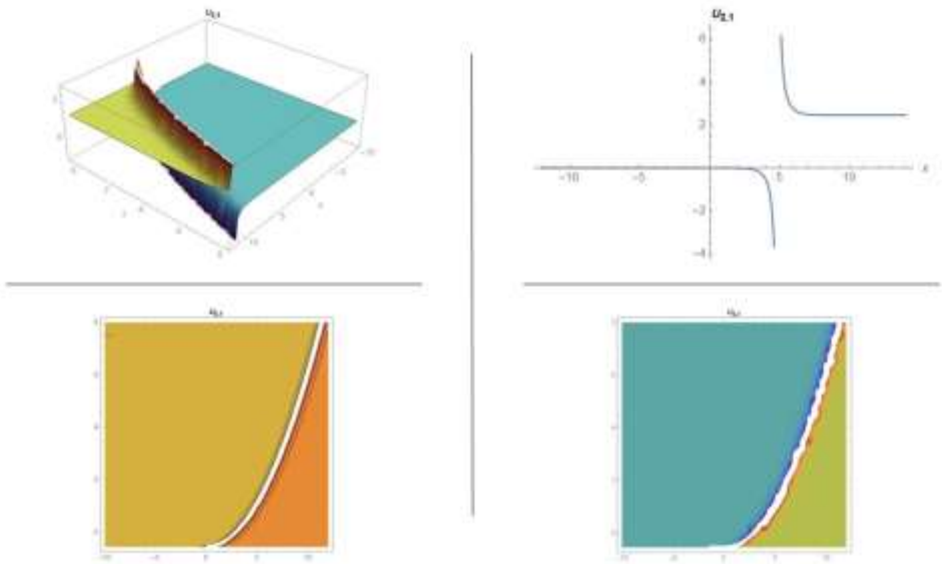

Figure 13. Display of Effective Graphs Showing the Behavior of Equation (42) as a Function of the Parameters $a_1 = -2.70492, q = 1.22, r = 0.75, s = 0.44, a_0 = 1.35246, a_2 = 1.35426, b_0 = 0.55, b_1 = -1.1, c = 1.94, \beta = 0.5, E = 0.75, t = 1.$

Conclusions

This study used two methods, MEFM and GKM, to obtain solutions to the Time Fractional Clannish Random Walker's parabolic equation. It has been found that both methods have superior and inferior aspects when examining the operation of these methods and the resulting solution functions. The similarity between the two methods is that the solutions are partly periodic. The most important advantage of using such functions is that the model continues to behave the same at each interval. The MEFM actually offers more options for varying the solving function. But this method has its own disadvantage. Because of the family 4 and family 5 constraints in particular, the solution functions may not appear in some instances. It is also possible to use this method to find solution functions for a nonlinear mathematical model and to produce diagrams showing how these solution functions behave. In conclusion, it can be said that both of these methods are effective techniques for obtaining the solutions of NPDEs that are of both the integer order and the fractional derivative type.

Authors' Contributions

All authors read and approved the final manuscript.

Ethic

There are no ethical issues with the publication of this article.

Conflicts of Interest

The authors declare that they have no competing interests.

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